

Soliton dynamics in a diffracting trapping potential

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Abstract: We examine a novel regime of interaction between a soliton and a transient trapping potential experimentally and numerically. We find that a soliton can be guided by such a potential, and arrested at its center by radiation damping.

OCIS codes: (190.6135) Spatial solitons; (190.5330) Photorefractive optics

Spatial solitons have been shown in the last decades to display rich physical phenomena, including stability in discrete lattices [1], self-healing [2], attraction and repulsion [3], and many other interesting behaviors. One line of investigation is concerned with the interaction of a soliton with a single waveguide. Solitons are attracted to the center of the waveguide and have been observed being trapped by a waveguide [4], escaping one [5,6], and being scattered by it [4]. Here we address the problem of interaction between a soliton and the 'z-dependent' potential of a rapidly expanding waveguide (where z is the direction of propagation). We derive an analytical model for the dynamics of the soliton, and demonstrate in both simulations and experiments the trapping of a spatial soliton is possible even in the extreme case when the interaction length of the soliton and the waveguide is of the order of the soliton formation time. Experiments are carried out in a photorefractive crystal (SBN:60) using the optical induction technique [1,7]. Our experimental scheme suggests a new regime of soliton dynamics in z -dependent potentials.

The physical picture can be understood in light of the well-established particle-like behavior of solitons which move in a refractive index analogously to a particle moving in a potential well with the propagation axis, z , replacing the role of time, and the refractive index playing the role of the potential in the paraxial Schrodinger equation (eq.(1)) [3]. The soliton beam (the "probe") is launched at the center of the z -dependent potential well, as portrayed in fig. 1a. The potential well is created by launching a "pump" beam in the opposite polarization of the soliton beam, which acts to induce a refractive index change in which the soliton propagates. Since the pump beam diffracts in z , the probe will experience both forcing and radiation damping.

The probe feels a positive saturable nonlinear contribution from the total intensity (of both the probe, the pump and of a uniform background illumination). The resulting model is the effective 2d+1 Schrödinger equations for the pump electric field amplitude ν and the probe electric field amplitude ψ ,

$$\begin{aligned} i\partial_t \psi(\bar{x}, t) &= -\frac{1}{2k_0} \Delta \psi(\bar{x}, t) + \frac{k_0}{n_0} n_1 \left(|\nu(\bar{x}, t)|^2 + \frac{|\psi(\bar{x}, t)|^2}{1 + g|\psi(\bar{x}, t)|^2} \right) \psi(\bar{x}, t) \\ i\partial_t \nu(\bar{x}, t) &= -\frac{1}{2k_0} \Delta \nu(\bar{x}, t) \end{aligned} \quad (1)$$

Here, $k_0 = \frac{2\pi n_0}{\lambda}$ is the wavenumber, $\lambda = 514nm$ is the wavelength of both laser beams, $n_0 = 2.34$ is the homogenous index of refraction in the photorefractive media (SBN:60), n_1 is the nonlinearity strength, and g is the nonlinear saturation constant. The parameters n_1 and g are well-known constants associated with the photorefractive crystal, the background illumination, and the voltage applied across the crystal and for our experimental parameters the total refractive index modulation is $\sim 3 \cdot 10^{-4}$.

In the experiments, we injected a probe beam with a tilt relative to a diffracting pump beam, focused on the input facet of the crystal to spots of diameter $\sim 8\mu m$ (probe) and $\sim 16\mu m$ (pump). We then altered the intensity of the pump beam (shown in profile in fig 1a), thereby altering the strength of the expanding waveguide. We measured the intensity of the probe beam at the output facet of the crystal (5mm and many Rayleigh lengths away from the input facet) at several pump intensities. The results of varying the pump intensity are shown in fig. 1b, which demonstrate the transition from unperturbed soliton formation at low pump intensities to strong trapping at high intensities.

Furthermore, we have found that when the initial beam is tilted (given an initial transverse velocity), it remains trapped until a critical velocity in which it escapes the trapping potential.

Motivated by the analysis in ([10],[11]), this behavior can be described in terms of a dynamical equation that we have derived to explain the motion of the soliton:

$$\ddot{\vec{x}} = -k_0 n_1 \nabla |v|^2 / n_0 - \gamma \dot{\vec{x}}$$

where \vec{x} is the position of the center of the soliton, and γ is a friction coefficient induced as a result of radiation loss. This model qualitatively exhibits all of the phenomena we observe in the experiment, such as trapping at the origin as a result of radiation-induced friction (fig. 1a, 1b), as well as the escape of the soliton for sufficient initial tilt. Furthermore, γ may be derived from the system parameters. Fig. 1c shows the simulated trajectory of the soliton in the trapped potential, for increasing values of the pump to probe ratio. This demonstrates trapping of the soliton in the expanding pump beam.

In sum, we have examined a novel regime of interaction between a soliton and a transient, diffracting potential, both computationally and experimentally, demonstrating that a soliton may become trapped in a forced and damped z-dependent system. This represents a novel and highly non-trivial regime of soliton transport.

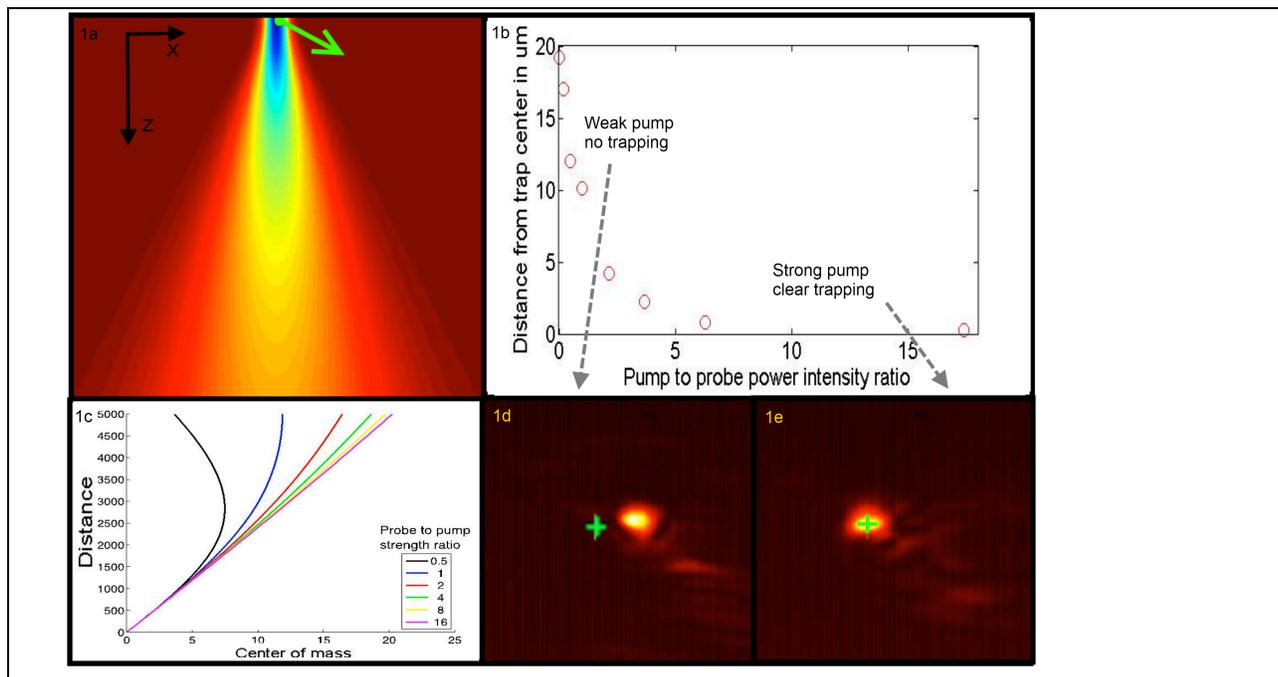


Fig. 1 – (a) A schematic of the diffracting Gaussian waveguide with the location and tilt of the probe beam marked by the green arrow. (b) The dislocation induced by the waveguide structure as a function of its strength. (c) Simulation results showing trapping as a result of increasing pump to probe intensity, over the course of propagation through the sample (all units in μm). (d-e) Experimental results at the output facet of the photorefractive crystal for a weak and strong pump beam accordingly. The green cross indicates the focus of the pump and expected center of trapping.

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